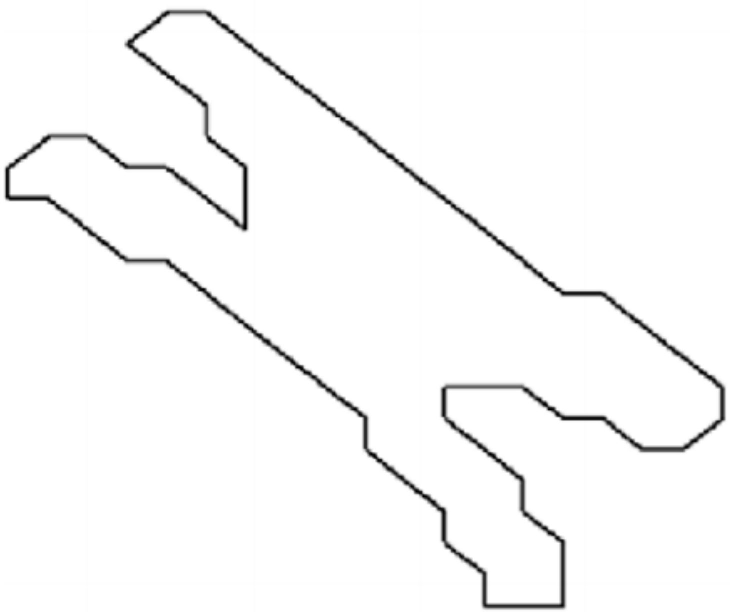
Complex networks in computer vision

Pavel Voronin

Shape to Network



[Backes 2009], [Backes 2010a], [Backes 2010b]

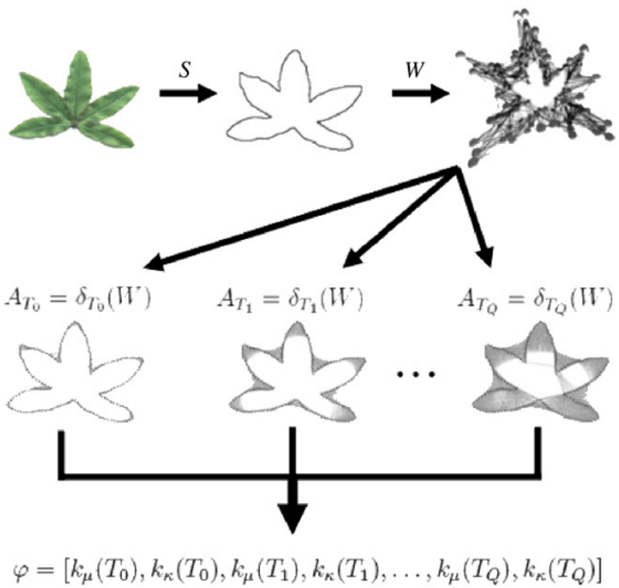
Weights + thresholds

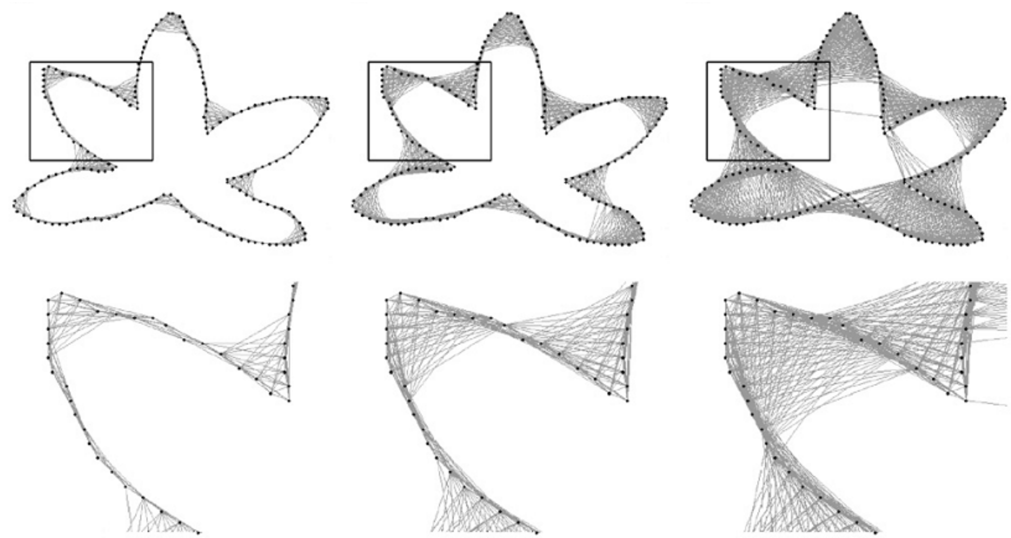
$$d(s_i, s_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}.$$

$$w_{ij} = W([w_i, w_j]) = d(s_i, s_j),$$

$$W = \frac{W}{\max_{w_{ij} \in W}}.$$

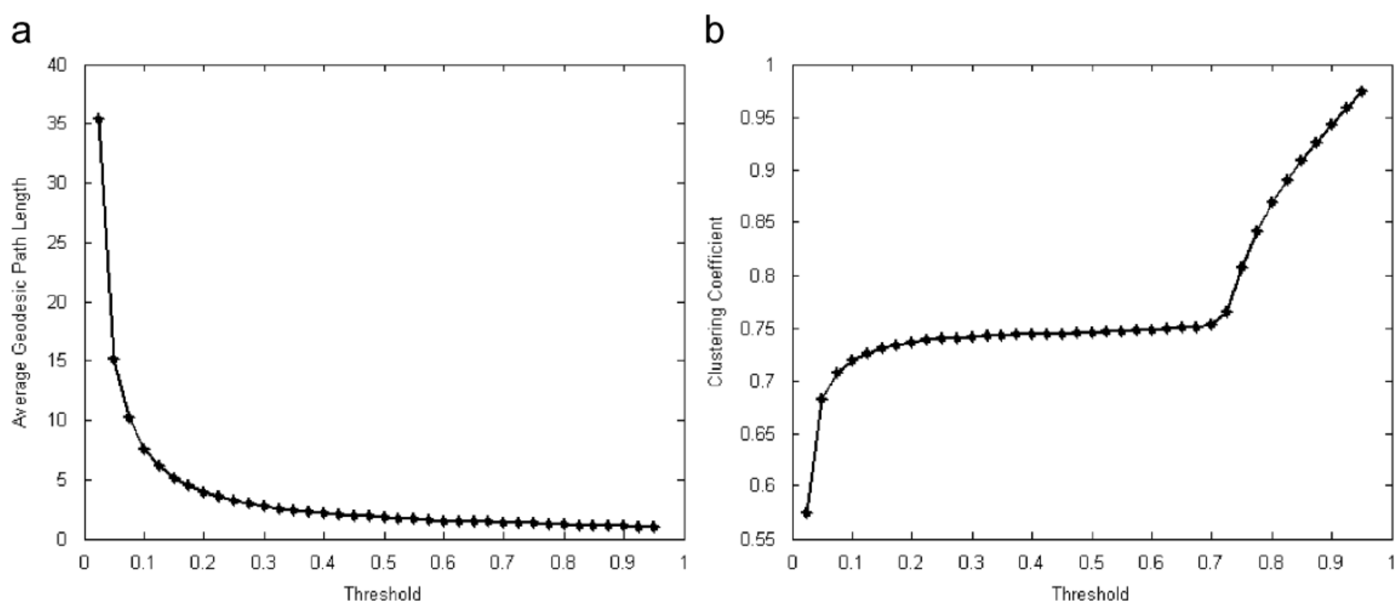
$$A_{T_l} = \delta_{T_l}(W) = \forall w \in W \begin{cases} a_{ij} = 0 & \text{if } w_{ij} \geqslant T_l, \\ a_{ij} = 1 & \text{if } w_{ij} < T_l. \end{cases}$$





Feature vectors

Features = characteristics of thresholded undirected binary networks: degree or joint degree (avg / min / max), avg path length, clustering coefficient, etc.



Multiscale Fractal Dimension

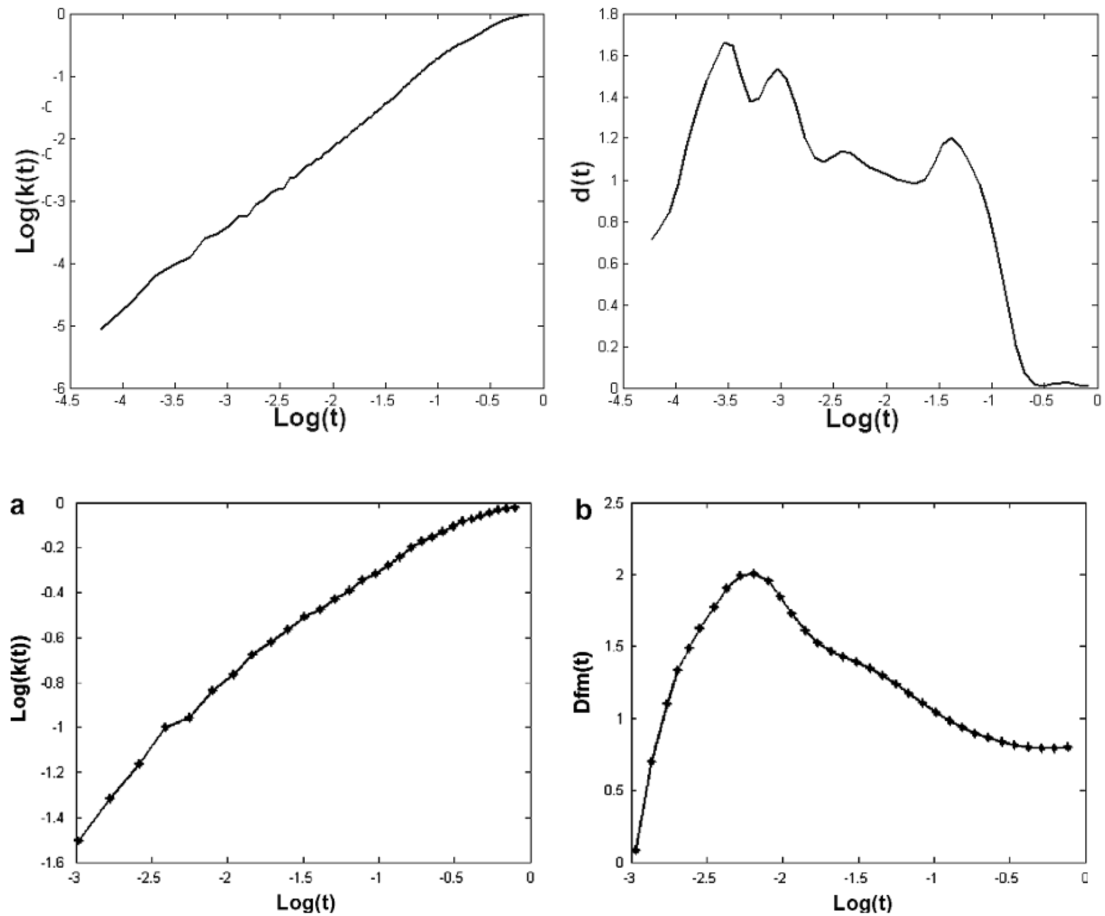
$$k \approx l^d$$

fractal dimension

$$d = \lim_{t \to 0} \frac{\log k(t)}{\log t}$$

Multi-Scale Fractal Dimension

$$d(t) = \frac{d \log k(t)}{d \log t}$$



Properties

- Only uses distances

  => rotation invariant

- Weights normalized by max distance

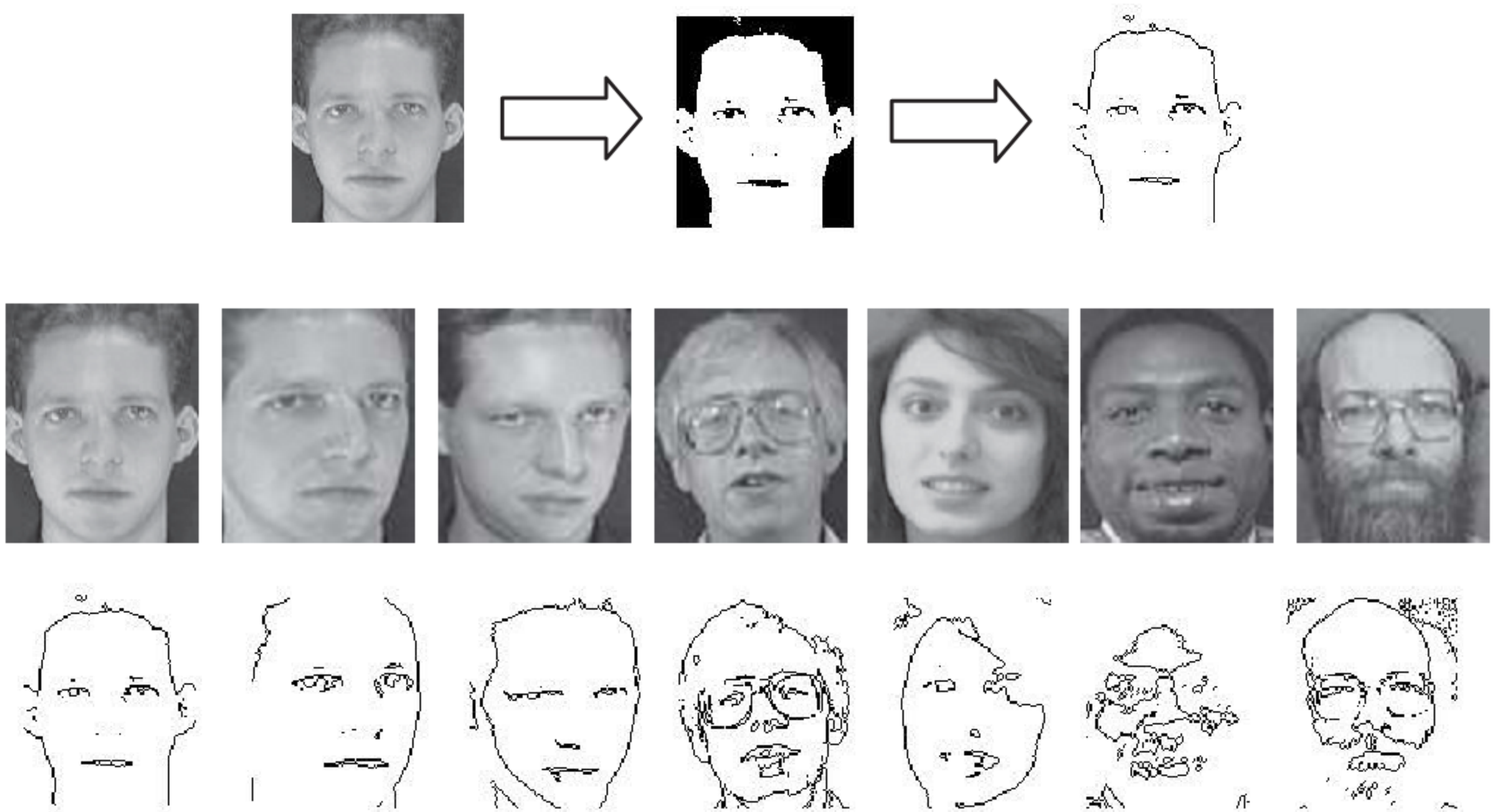
  => scale invariant

- Uses pixels not curve elements

  => robust to noise and outliers

  => applicable to skeletons, multiple contours

Face recognition



[Goncalves 2010], [Tang 2012a]

Multiple binarization thresholds

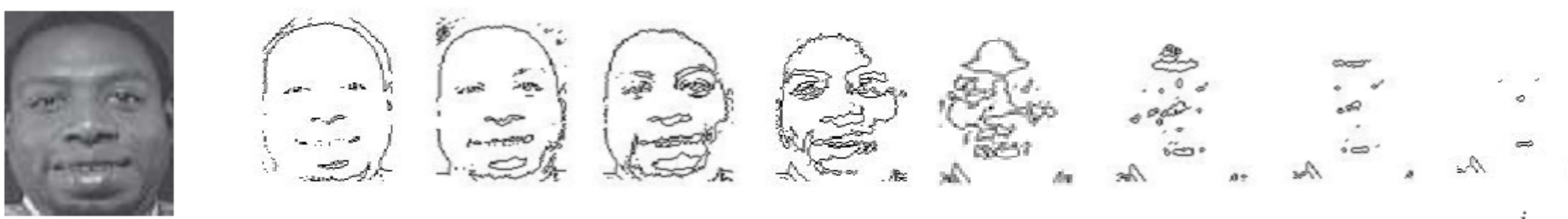


Figure 4. Image contours of black people with different values of t^i

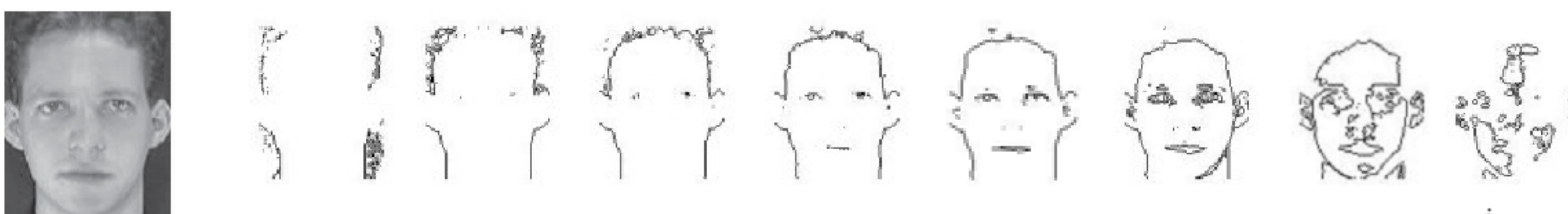


Figure 5. Image contours of white people with different values of t^i

Texture to Network

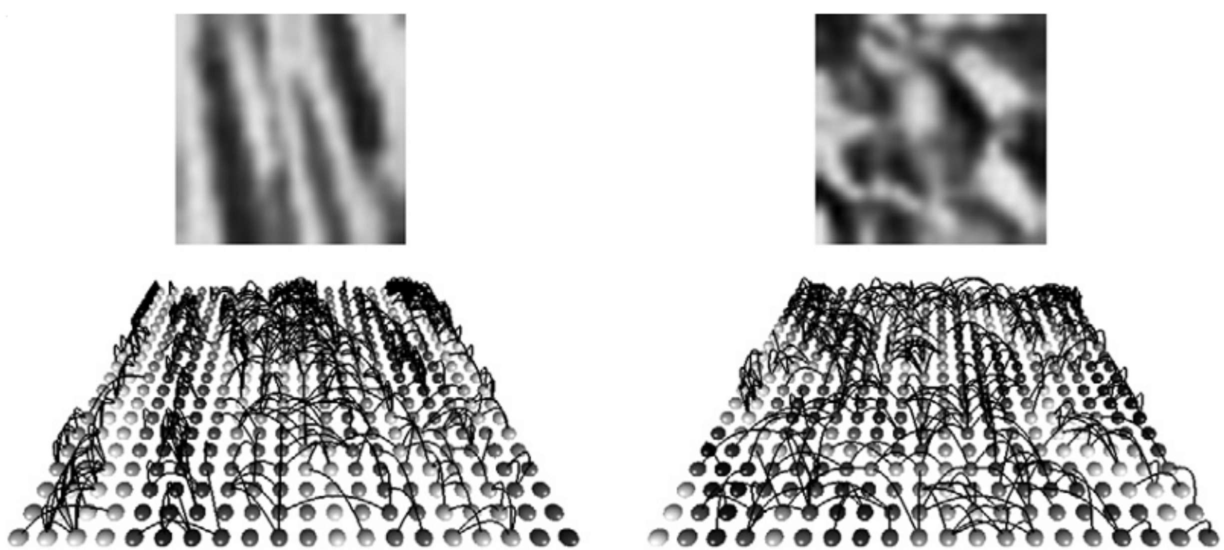
$$E = \left\{ e = (v_{x,y}, v_{x',y'}) \in I \times I \middle| \sqrt{(x - y')^2 + (x - y')^2} \leqslant r \right\}.$$

$$w(e) = (x - x')^2 + (y - y')^2 + r^2 \frac{|I(x,y) - I(x',y')|}{L}$$

44	31	31	29	35	103	39	29	31
25	23	27	21	42	91	56	20	32
28	18	21	37	69	56	49	21	34
82	20	52	140	70	40	44	30	33
113	17	45	155	52	44	50	35	31
95	20	12	20	58	129	26	32	36
72	28	28	14	60	52	39	34	35
38	15	13	17	53	62	40	27	37
18	16	14	10	38	65	39	26	35

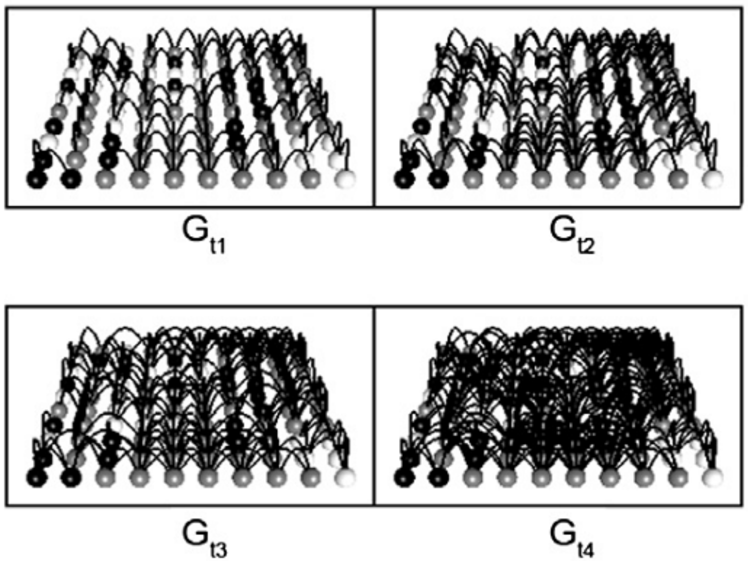
44	31	31	29	35	103	39	29	31
25	23	27	21	42	91	56	20	32
28	18	21	37	69	56	49	21	34
82	20	52	140	70	40	44	30	33
113	17	45	155	52	44	50	35	31
95	20	12	20	58	129	26	32	36
72	28	28	14	60	52	39	34	35
38	15	13	17	53	62	40	27	37
18	16	14	10	38	65	39	26	35

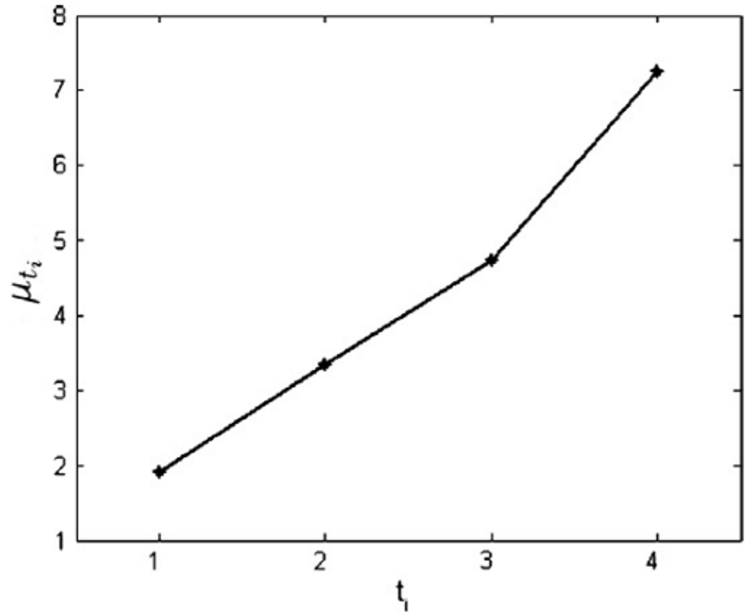
				0,52				
		0,51	0,31	0,26	0,29	0,45		
		0,28	0,28	0,09	0,13	0,29		
	0,57	0,24	0,26		0,07	0,23	0,53	
		0,36	0,17	0,07	0,26	0,33		
		0,49	0,35	0,24	0,28	0,47		
				0,50				



[Chalumeau 2008], [Backes 2010c], [Backes 2013]

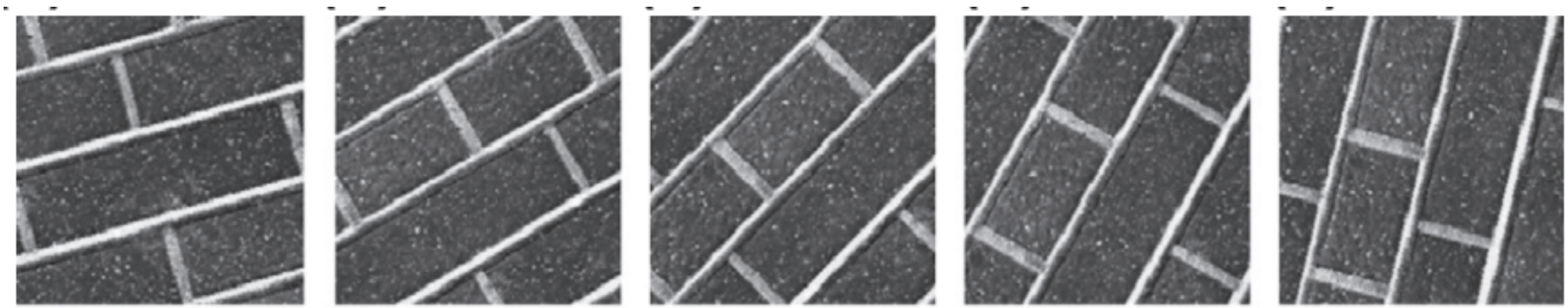
Thresholds + features

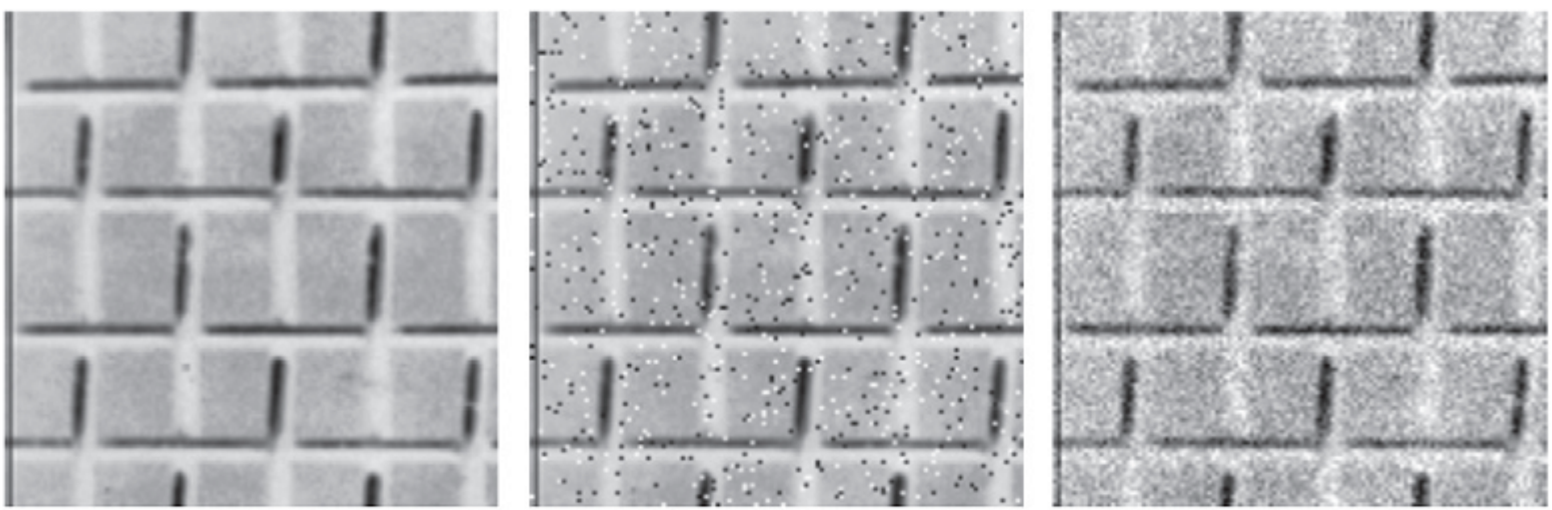




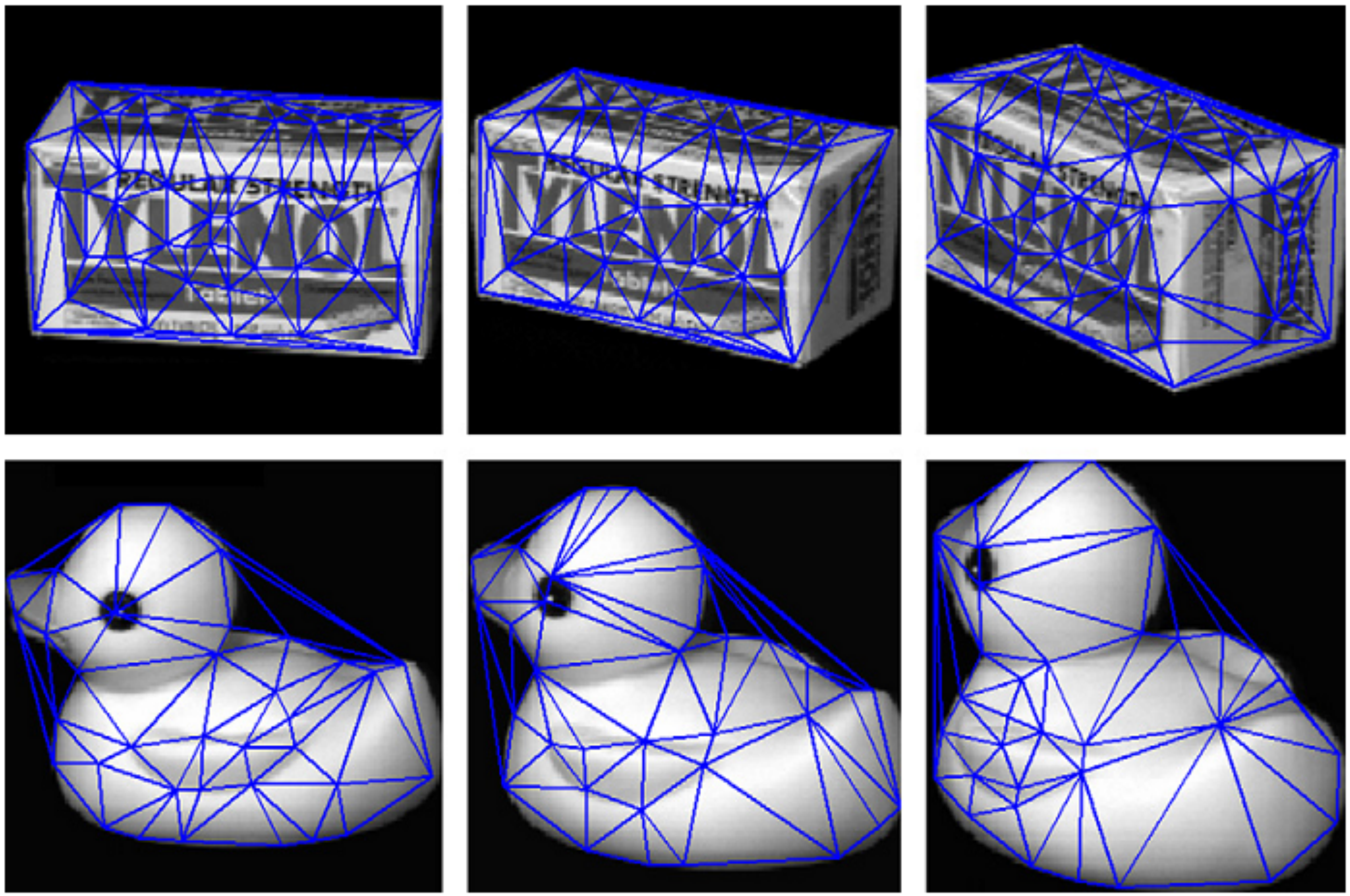
Features: degree, hierarchical degree (avg / min / max)

Invariance, robustness



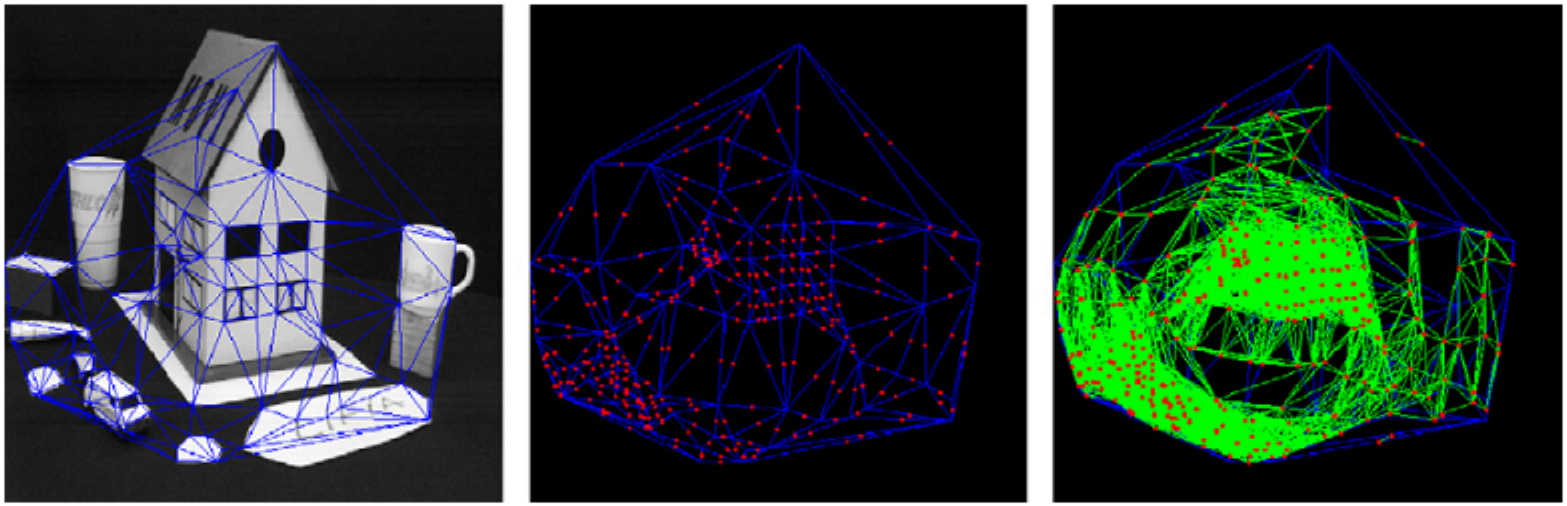


Graph structure analysis



[Tang 2012b]

Graph to Network



$$w_{ij} = d(e_i, e_j)$$

$$w_{ij} = W\left([e_i, e_j]\right)$$

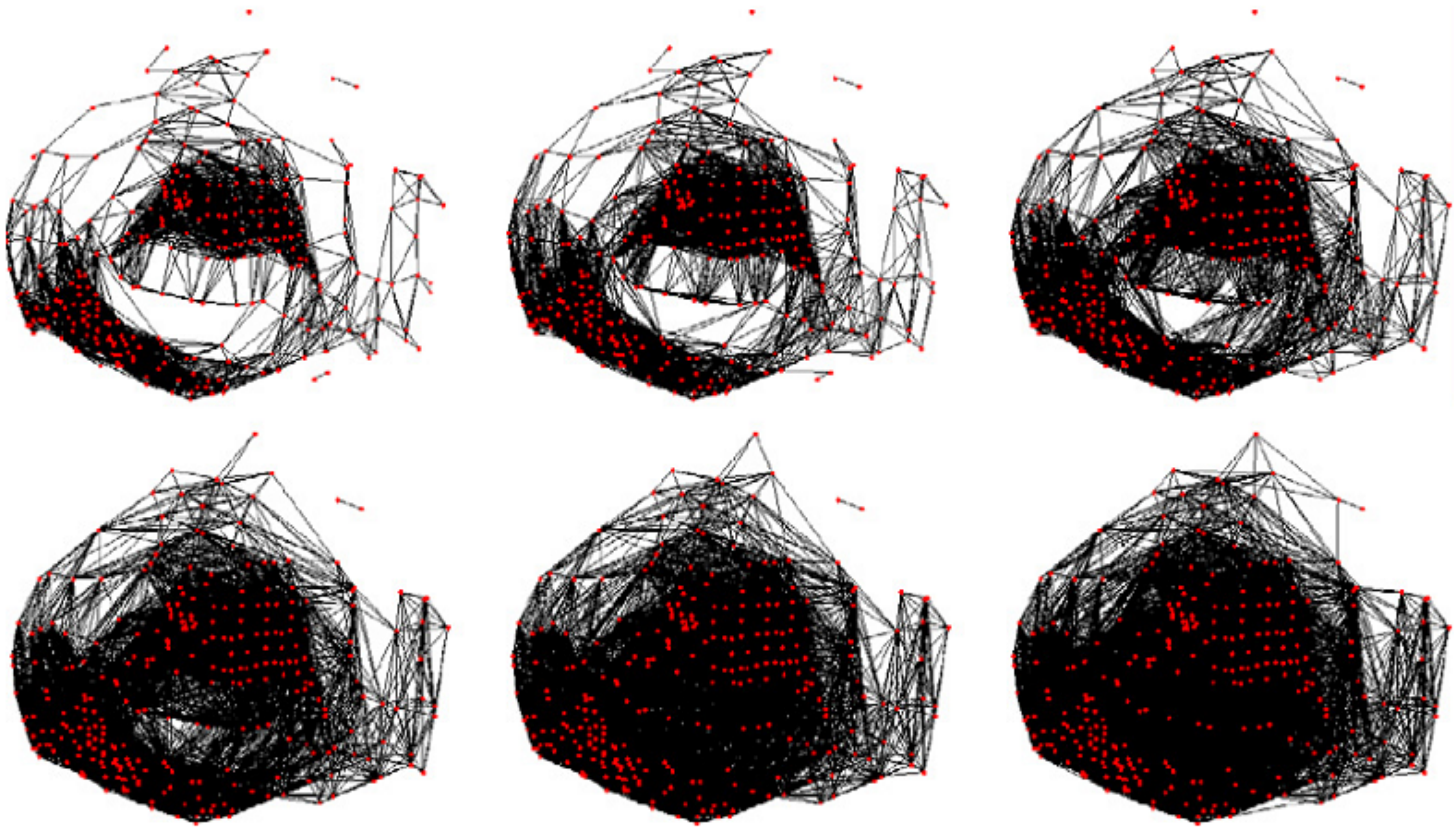
$$W = \frac{W}{\max_{w_{ij} \in W}}$$

$$e_i = (l_i, d_i, d_{1i}, d_{2i}, x_i, y_i),$$

where

l_i: length of the edge,
d_i: distance between the edge center and the center of the graph, O,
d_{1i}, d_{2i}: distances from the beginning and end of the edge, respectively, to graph center, O,
x_i, y_i: coordinates of the center point of the edge.

Thresholds + descriptors

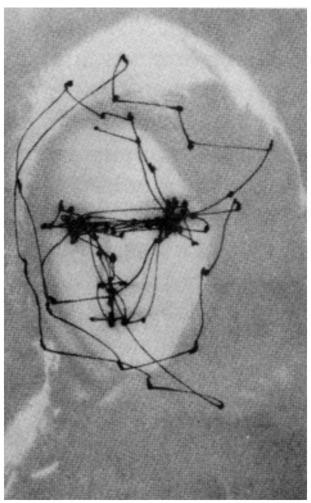


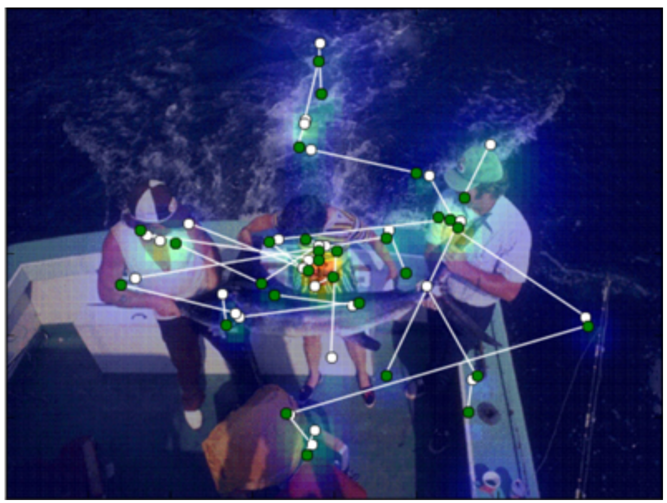
Degree, joint degree, clustering-distance

Saliency

Saccade eye movements + different fixation time = saliency map

=> model them as walks in networks





[Harel 2006], [Costa 2007], [Gopalakrishnan 2010], [Pal 2010], [Kim 2013]

Network construction

Nodes

- pixels
- segments
- blocks

Edges

- local (neighbourhood)
- global (most similar)
- both

Edge weights = (distance in feature space) / (distance in image space)

Features:

- relative intensity
- entropy of local orientations
- compactness of local colour

Random walks

Eigenvector centrality

== stationary distribution for Markov chain

== expected time a walker spends in the node

$$\mathbf{A}(j,i) = w_{ij}$$

$$\mathbf{W}(i,i) = \sum_j w_{ij}$$

$$\mathbf{P} = \mathbf{A}\mathbf{W}^{-1}$$

$$\boldsymbol{\pi} = \mathbf{P}\boldsymbol{\pi}$$

saliency

$$s_i = \frac{\boldsymbol{\pi}(i) - \pi_{\min}}{\pi_{\max} - \pi_{\min}}$$

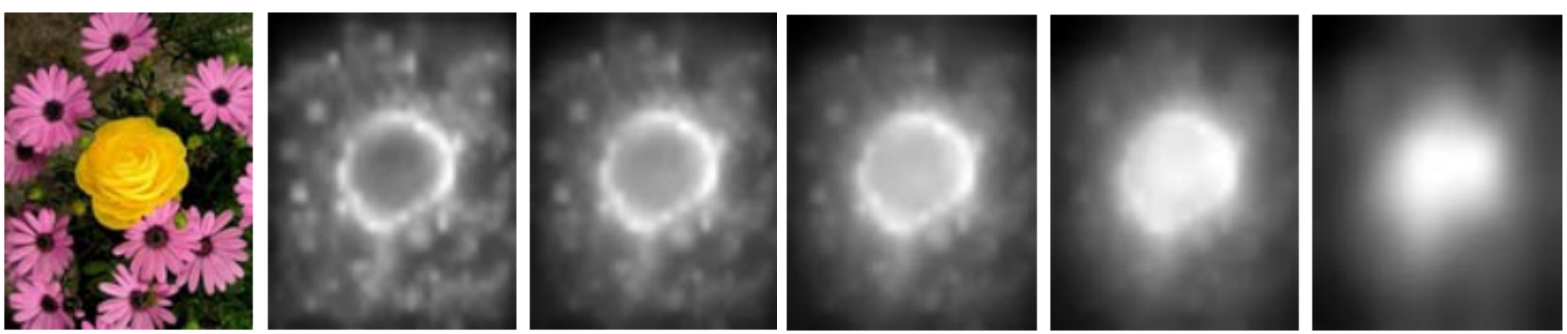
Random walk with restart (RWR)

If we have prior info on node importance, we can use RWR.
After finishing a walk, agent restarts it with some probability, choosing new starting node according to an importance map.

The stationary distribution $\mathbf{r}_k$ satisfies

$$\mathbf{r}_k = (1-\epsilon)\,\mathbf{P}\mathbf{r}_k + \epsilon\mathbf{e}_k$$

$$\begin{aligned}\mathbf{r}_k &= \epsilon\left(\mathbf{I} - (1-\epsilon)\,\mathbf{P}\right)^{-1}\mathbf{e}_k \\ &= \mathbf{Q}\mathbf{e}_k,\end{aligned}$$

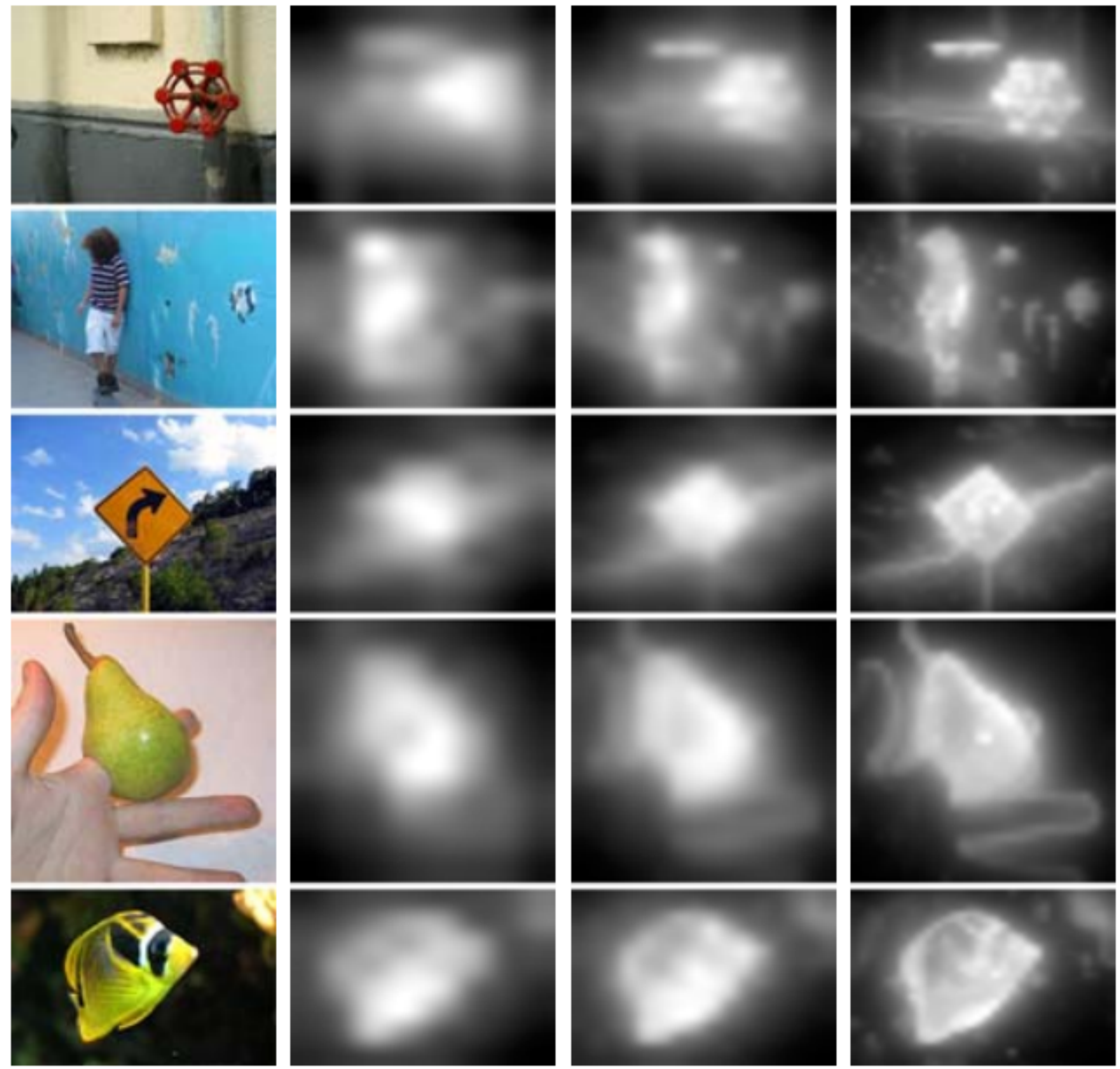


varying restarting probabilities

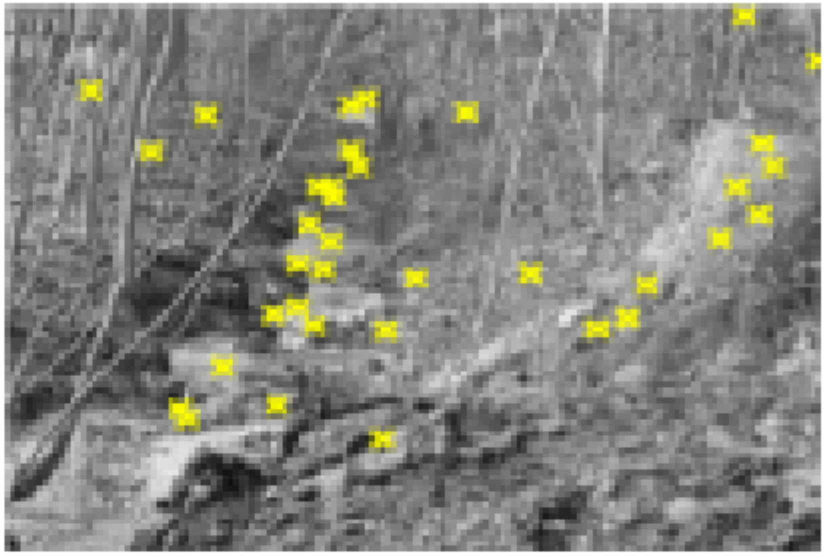
RWR for Multiscale estimation

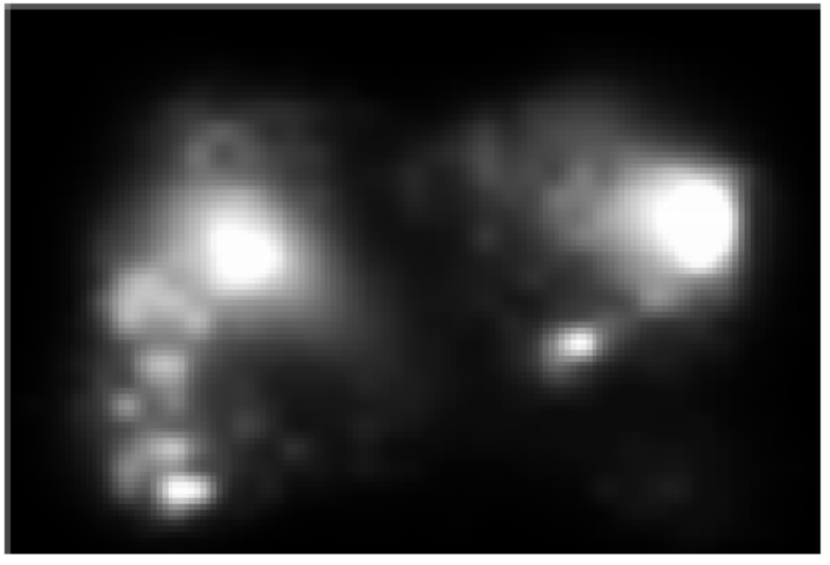
Using distribution from
a coarser scale:

$$\mathbf{r}_\pi^{(l)} = (1-\epsilon)\mathbf{P}^{(l)}\mathbf{r}_\pi^{(l)} + \epsilon U(\mathbf{r}_\pi^{(l-1)})$$



Compare to real data





Interest points 1

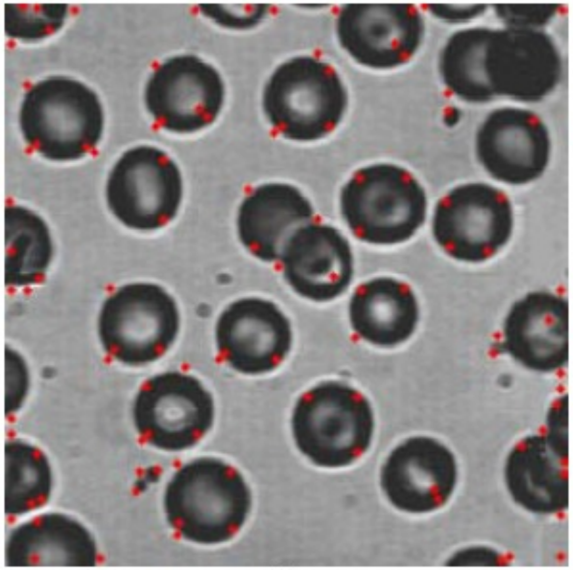
nodes = segments

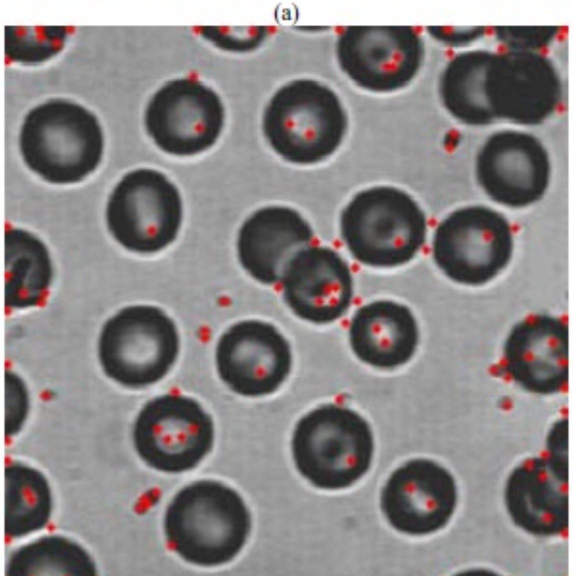
$$s(i) = \sum_{(i,j)\in E} w(i,j)$$

$$w(p_i, p_j) = \begin{cases} |f(p_i) - f(p_j)| & \text{if } r_i \text{ and } r_j \text{ are adjacent} \\ 0 & \text{otherwise.} \end{cases}$$

[Criado 2011]

Interest points 2


(a)


(b)

$I(x),\ I(y) \in M_{N\times N}$

gradient of the image :

$\omega(I_{ij}, I_{mn}) = | I_{ij} - I_{mn} |$

sliding node window W_{nodes}

$$\omega_{ij} = \sum_{I_{mn} \in W_{nodes}} \omega(x)(I(x)_{ij}, I(x)_{mn}) * \omega(y)(I(y)_{ij}, I(y)_{mn})$$

$$\begin{cases} d_{ij} = \omega_{ij}, \text{ if } d_{ij} > \xi\ \omega_{\max} \text{ and } d_{ij} > d_{mn} \\ d_{ij} = 0, \text{ otherwise.} \end{cases}$$

Suppress non-interest points for:

$$\begin{cases} |d_{ij} - d_{pq}| < \xi\ d_{\max} \\ \sqrt{(p-i)^2 + (q-j)^2} < T \end{cases} ,\quad i, j, p, q \in (1, N)$$

[Zhang 2012]

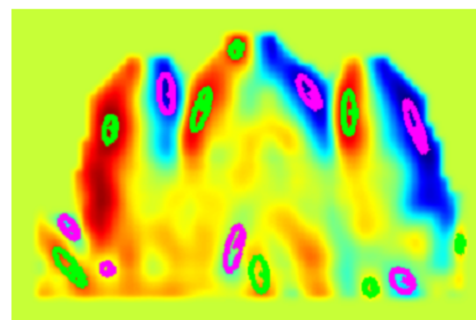
Interest points 3

G

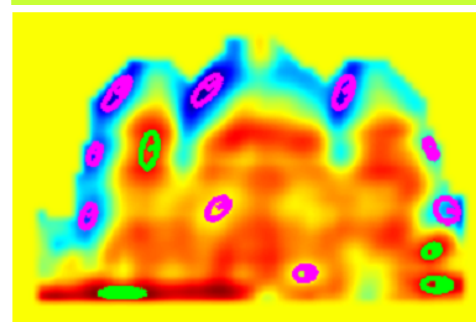
Joint graph:
G(V, E, W)
V – vertices: all pix in both images
E – edges: all possible (complete)
W – edge weights: affinity matrix

$W(x,y) = \exp(-\|f(x)-f(y)\|^2/\sigma_f^2)$

f – descriptor (e.g. SIFT)



J



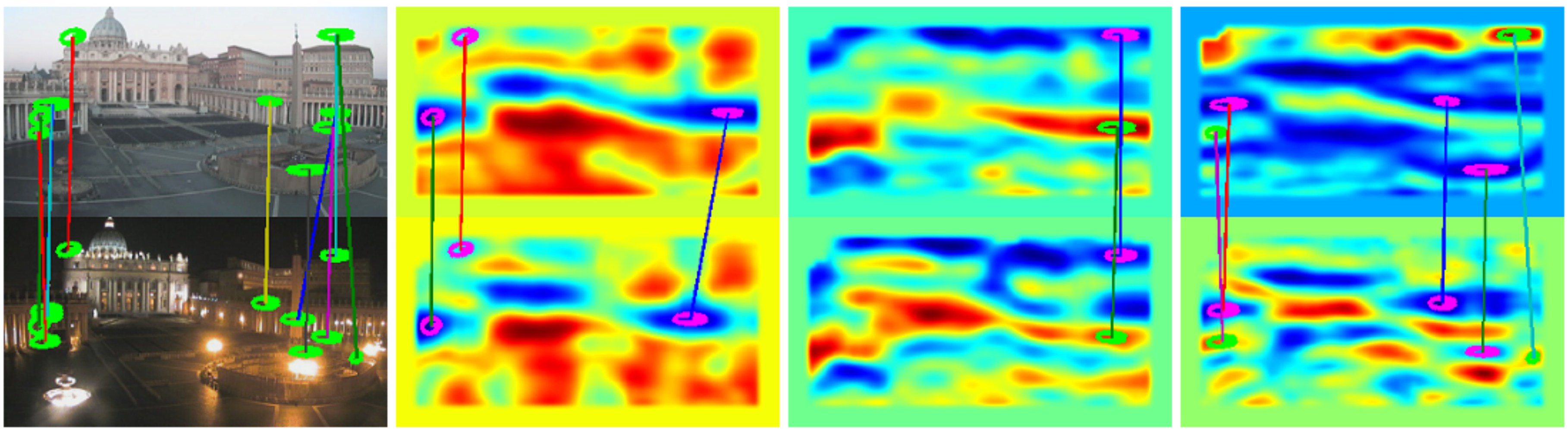
U – first *N* eigen-vectors of *W*
J, K – maps of *U* in image-spaces

Interest points = stable extrema of *J* or *K*

(using MSER blob-detector)
(Maximally Stable Extremal Region)

[Bansal 2013]

Sample result



[Bansal 2013]

Large networks: libraries

- **NetworkX** (python) http://networkx.github.io
- **Boost Graph Library** (C++) http://www.boost.org/libs/graph/
- **MatlabBGL** (sparse matrices + Boost) http://dgleich.github.io/matlab-bgl/
- **PEGASUS** (Hadoop = MapReduce) http://www.cs.cmu.edu/~pegasus/

Large networks: centrality

- **Approximate betweenness** [Brandes 2006]

Estimated from a limited number of single-source shortest-paths computations.

- **K-path centrality** [Kourtellis 2012]

Message traversals from all possible source nodes along random simple paths of at most K edges.

- **HyperLogLog counters** [Boldi 2013]

Geometric centralities on very large graphs estimated in a semi-streaming fashion.

Bibliography (1 / 3)

[Backes 2009] Backes, Casanova, Bruno - 2009 - *A complex network-based approach for boundary shape analysis*

[Backes 2010a] Backes, Bruno - 2010 - *Shape Skeleton Classification Using Graph and Multi-scale Fractal Dimension*

[Backes 2010b] Backes, Bruno - 2010 - *Shape classification using complex network and Multi-scale Fractal Dimension*

[Backes 2010c] Backes et al - 2010 - *A Complex Network-Based Approach for Texture Analysis*

[Backes 2013] Backes et al - 2013 - *Texture analysis and classification: A complex network-based approach*

[Bansal 2013] Bansal, Daniilidis - 2013 - *Joint Spectral Correspondence for Disparate Image Matching*

Bibliography (2 / 3)

[Boldi 2013] Boldi, Vigna - 2013 - *In-Core Computation of Geometric Centralities with HyperBall: A Hundred Billion Nodes and Beyond*

[Brandes 2006] Brandes, Pich - 2006 - *Centrality Estimation in Large Networks*

[Chalumeau 2008] Chalumeau et al - 2008 - *Texture Discrimination Using Hierarchical Complex Networks*

[Costa 2007] da Fontoura Costa - 2007 - *Visual Saliency and Attention as Random Walks on Complex Networks*

[Criado 2011] Criado et al - 2011 - *Interest Point Detection in Images by a Local Centrality Algorithm on Complex Networks*

[Goncalves 2010] Goncalves et al - 2010 - *A Rotation Invariant Face Recognition Method Based on Complex Network*

[Gopalakrishnan 2010] Gopalakrishnan et al - 2010 - *Random Walks on Graphs for Salient Object Detection in Images*

Bibliography (3 / 3)

[Harel 2006] Harel et al - 2006 - *Graph-Based Visual Saliency*

[Kim 2013] Kim et al - 2013 - *Multiscale Saliency Detection Using Random Walk with Restart*

[Kourtellis 2012] Kourtellis et al - 2012 - *Identifying High Betweenness Centrality Nodes in Large Social Networks*

[Pal 2010] Pal et al - 2010 - *Modelling visual saliency using degree centrality*

[Tang 2012a] Tang et al - 2012 - *A rotation and scale invariance face Recognition Method Based on Complex Network and Image Contour*

[Tang 2012b] Tang et al - 2012 - *Graph structure analysis based on complex network*

[Zhang 2012] Zhang et al - 2012 - *A Complex Network-Based Approach for Interest Point Detection in Images*